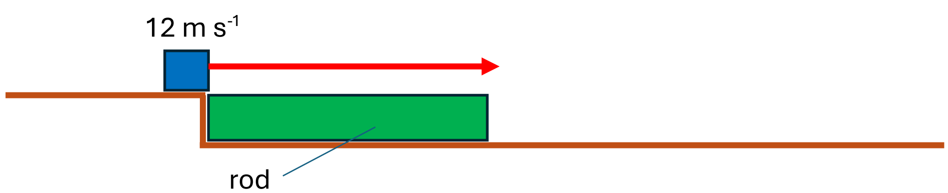
Teacher notes

Topic A

A challenging problem involving friction and momentum conservation.

A block of mass 2.0 kg moves with initial speed u = 12 m s^{-1} as it begins to slide over a rod of length 7.2 m and mass 4.0 kg that is initially at rest on a frictionless surface. The coefficient of dynamic friction between the rod and the block is μ.

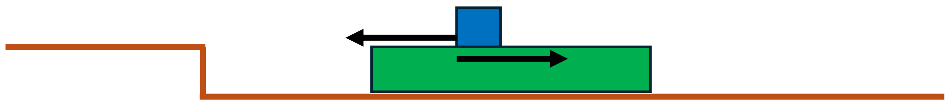


When the block gets to the right-hand end of the rod the block and the rod move with the same speed v. Take g = 10 m s^{-2}.

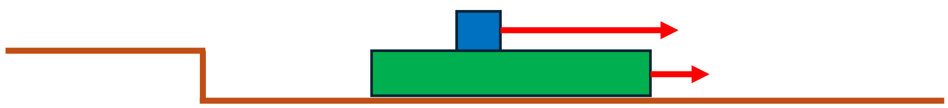
(a) Explain why the rod will start moving to the right.
(b) Calculate, in terms of μ the acceleration of the rod and that of the block while the block is sliding over the rod.
(c) Explain why momentum conservation can be applied to the block-rod system.
(d) Calculate the common speed v.
(e) Determine μ.

Answers

(a) There is a frictional force exerted on the block opposing its motion and so by Newton's third law a force of equal magnitude acts on the rod to the right, accelerating the rod to the right.



The block is slowing down, and the rod is accelerating:

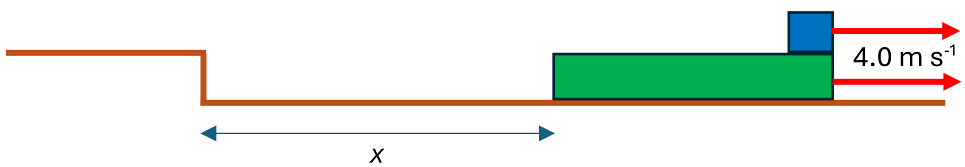


(b) The frictional force is $f = \mu mg = 20\mu$ and so the acceleration of the rod is $a = \frac{f}{M} = \frac{20\mu}{4.0} = 5\mu$. The (deceleration) of the block is $a = \frac{f}{m} = \frac{20\mu}{2.0} = 10\mu$.

(c) There is no external force acting on the block-rod system. The frictional force on the block is equal and opposite to the frictional force on the rod.

(d) Applying momentum conservation we find: $2.0 \times 12 = (2.0 + 4.0)v \quad \Rightarrow \quad v = 4.0 \text{ m s}^{-1}$. This is the common speed of the rod and block when the block gets to the end of the rod.

(e) By this time the rod moved a distance *x* to the right.



We can obtain μ using the always powerful work-kinetic energy relation. The net work is the change in kinetic energy and therefore

$$W_{net} = \Delta K$$

$$\Delta K = \frac{1}{2}(m+M)v^2 - \frac{1}{2}mu^2 = \frac{1}{2} \times 6.0 \times 4.0^2 - \frac{1}{2} \times 2.0 \times 12^2 = -96 \text{ J}$$

The net work is done by the friction force acting on the block $-f(x+7.2)$ and the friction force acting on the rod $+fx$:

$$W_{net} = -f(x+7.2)+fx = -f \times 7.2 = -\mu mg \times 7.2 = -144\mu$$

Hence

$$144\mu = 96 \quad \Rightarrow \quad \mu \approx 0.667$$

Equivalently, we can use kinematics. Applying kinematics to the rod:

$v^2 = 0 + 2a_{rod}x = 2 \times 5\mu \times x = 10\mu x$, i.e. $16 = 10\mu x$.

The block moved a distance 7.2 + x so applying kinematics to the block we find:

$v^2 = 12^2 - 2a_{block}(x+7.2) = 144 - 2 \times 10\mu \times (x+7.2)$. Equating the two speeds we get:

$$10\mu x = 144 - 20\mu(x+7.2)$$

Solving for x we get:

$$30\mu x = 144 - 20\mu \times 7.2$$

$$x = \frac{144-144\mu}{30\mu}$$

$$x = 4.8 \times \frac{1-\mu}{\mu}$$

Hence from $16 = 10\mu x$ we find:

$$16 = 10\mu \times 4.8 \times \frac{1-\mu}{\mu} = 48(1-\mu)$$

Hence,

$$1-\mu = \frac{16}{48} = \frac{1}{3}$$

$$\mu = \frac{2}{3} \approx 0.667$$

The slight advantage of the kinematics approach is that we can also find that the distance travelled by the rod until it has the same speed as the block is

$$x = 4.8 \times \frac{1-\mu}{\mu} = 4.8 \times \frac{1-\frac{2}{3}}{\frac{2}{3}} = 2.4 \text{ m.}$$